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A NOTE ON PARTIAL CAVITATION OF FLAT PLATE HYDROFOILS

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Introduction

Recently Tulin[1] and Wu[2] have treated the problem of fully developed cavitation on flat plate and cambered foils. In these treatments, the length of the cavity is always greater than the chord of the hydrofoil and the cavity is assumed to start at the leading edge of the plate. The purpose of this note is to extend Tulin's work to account for partial cavitation, i.e., when the cavitation bubble is less than the hydrofoil chord.

Notation

A, B. C. D	-	constants
a	-	cavity length parameter
c	-	chord
C_p	-	pressure coefficient = $p - p_\infty / \frac{1}{2} \rho U^2$
C_L	-	lift coefficient = lift/c $\frac{\rho}{2} U^2$
i	-	$\sqrt{-1}$
K	-	cavity number = $p_\infty - p_k / \frac{1}{2} \rho U^2$
ℓ	-	cavity length
p	-	pressure (p_∞ denotes free stream pressure, p_k cavity pressure)
U	-	free stream velocity
u	-	perturbation velocity in x direction
v	-	perturbation velocity in y direction
w	-	perturbation velocity function = $u - iv$
z	-	physical plane = $x + iy$
α	-	angle of attack
ξ, ζ	-	auxiliary planes
η	-	coordinate of hydrofoil

Statement of Problem

The cavity bubble is a surface of constant pressure. Thus, on the wetted portion of the hydrofoil the slope of the foil is specified, and on the cavity portion the magnitude of the velocity is specified. The flow is shown schematically in Fig. 1. The assumptions of thin airfoil theory will now be made, i.e., the slope, angle of attack, camber and thickness are small so that the boundary conditions can be written as follows: On the wetted portions

$$\text{(i)} \qquad v = \frac{d\eta}{dx} U ,$$

and on the cavity

$$\text{(ii)} \qquad u = \frac{K}{2} U \ .$$

Equation (i) follows from the linearized Bernoulli equation and the definition of cavitation number K. It is further required that the perturbation velocities u, v vanish at infinity, i.e.,

$$\text{(iii)} \qquad u, v \to 0 \ \text{ as } \ z \to \infty ,$$

and that the velocity be finite at the trailing edge. The final restriction is that the shape of cavity plus hydrofoil is closed. This condition is expressed mathematically as

$$\text{(iv)} \qquad \oint_{\text{body}} d\eta = 0 \ .$$

Method of Solution

The solution is effected by conformal mapping of the complex velocity function $u - iv$ and is the same as Tulin's procedure. The hydrofoil and cavity is regarded as a slit in the physical plane (z plane) and this slit is mapped onto auxiliary planes in such a way as to separate the cavity and wetted portions. Thus consider the sequence of mappings:

$$\zeta = a i \left(\frac{z+1}{z-1}\right)^{1/2} \tag{1}$$

and

$$\zeta = \frac{1}{2} + \frac{1}{4} \left(\xi + \frac{1}{\xi}\right) , \tag{2}$$

where

$$a = \left(\frac{2-\ell}{\ell}\right)^{1/2} = \left(\frac{c}{\ell} - 1\right)^{1/2} . \tag{3}$$

The relation between the various planes is shown in Fig. 2. Equation (1) maps the slit in the physical plane onto the real axis in the ζ plane in such a way the cavity placed on $R\ell\ 0 \leq \zeta \leq 1$ and the upper half ζ plane is mapped onto the entire z plane. The point $\zeta = ia$ corresponds to infinity in the z plane. Equation (2) transforms the cavity portion of the real axis in the ζ plane to the upper half of the unit circle in the ξ plane, the wetted portion corresponding to real $-1 \geq \xi \geq 1$. The points A and B correspond to the leading and trailing edges of the hydrofoil and point C designates the end of the bubble. In both auxiliary planes the trailing edge B is at infinity.

<u>Solution for Flat Plate Hydrofoil</u>

In this case Condition (i) can be written as

$$v = -U\alpha ,$$

where α is the angle of attack. In the ξ plane, therefore, u must be constant on the upper half of the unit circle and v must be a constant on the real axis. It is known that a source located at the points $\xi = \pm 1$ satisfies these conditions, and, in addition, gives a zero velocity at infinity. Thus in the ξ plane the complex velocity must be of the form

$$w(\xi) = u - iv = \frac{A}{\xi + 1} + \frac{B}{\xi - 1} + C + iD$$

where A, B, C, D are real constants. The introduction of the ξ plane was merely to illustrate the type and location of singularities required. The velocity function above can be transformed to the ζ plane where it takes the form

$$w(\zeta) = u - iv = \frac{A-B}{2} + \frac{1}{2}\left[B\sqrt{\frac{\zeta}{\zeta-1}} - A\sqrt{\frac{\zeta-1}{\zeta}}\right] + C + iD \tag{4}$$

Conditions (i) through (iv) can now be applied to determine the constants.

Application of Conditions (i) and (ii) give

$$\frac{KU}{2} = \frac{A-B}{2} + C \ ,$$

and

$$D = U\alpha \ .$$

At the point $\zeta = ia$, the perturbation velocities must vanish, which gives two additional relations:

$$A = \frac{(2a)^{1/2} U}{(a^2+1)^{1/4}} \left\{ \frac{k}{2(\cos\theta/2 + \sin\theta/2)} - \frac{\alpha}{\cos\theta/2 - \sin\theta/2} \right\}$$

$$B = \frac{-2^{1/2}(a^2+1)^{1/4} U}{a^{1/2}} \left\{ \frac{k}{2(\cos\theta/2 + \sin\theta/2)} + \frac{\alpha}{\cos\theta/2 - \sin\theta/2} \right\} \tag{5}$$

where $\theta = \frac{\pi}{2} + \tan^{-1}(1/a)$. Another relation between the constants A and B is provided by the closure requirement of Condition (iv). This condition can be written as

$$\oint_{\text{body}} dy = \oint_{\text{body}} v(x)\, dx = 0 \ ,$$

or

$$\text{Im} \oint_{\text{body}} w(\zeta) \frac{dz}{d\zeta}\, d\zeta = 0 \tag{6}$$

where Im denotes the imaginary part of the integral. As the boundary of the foil is traced out in the counterclockwise direction in the z plane, ζ proceeds along the real axis from positive to negative infinity. Thus, the following integral must be considered

$$J = \int_{\infty}^{-\infty} w(\zeta) \frac{4a^2 \zeta\, d\zeta}{(\zeta + ia)^2 (\zeta - ia)^2} \tag{7}$$

where $dz/d\zeta$ has been computed from Eq. (1) and $w(\zeta)$ is to be found from Eqs. (4), (5). The integrand of J contains double poles at $\zeta = \pm\, ia$ and is such that J can be evaluated by contour integration, i.e.,

$$J = -2\pi i \,(\text{Res. at } \zeta = ia) .$$

The result of these operations is

$$J = \frac{\pi a^{1/2} 2^{1/2} B}{4(a^2+1)^{3/4}} \left\{ \cos\frac{3\theta}{2} - \sin\frac{3\theta}{2} - i\left(\cos\frac{3\theta}{2} + \sin\frac{3\theta}{2}\right)\right\}$$

$$- \frac{\pi a^{-1/2} 2^{-1/2} A}{4(a^2+1)^{1/4}} \left\{ \cos\frac{\theta}{2} + \sin\frac{\theta}{2} + i\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)\right\} \tag{8}$$

The requirement that $\text{Im}\, J = 0$ gives the required additional relation between the constants A and B. This result, together with Eq. (5), is used to determine the constants, and so to obtain the relation between the cavity length ℓ, cavitation number K and angle of attack α, i.e.,

$$\frac{K}{2\alpha} = \frac{1}{\left(\frac{c}{\ell} - 1\right)^{1/2}} \left[2\frac{c}{\ell} - 1 + 2\left(\frac{c}{\ell}\right)^{1/2} \left(\frac{c}{\ell} - 1\right)^{1/2} \right] . \tag{9}$$

This equation is plotted in Fig. 3.

Calculation of the Lift

If the usual limitations of thin airfoil theory are observed, the lift coefficient can be determined from the approximate formula

$$C_L = -2 \oint_{\text{body}} \frac{u}{U}\, d\left(\frac{x}{c}\right) . \tag{10}$$

Thus

$$C_L = -\frac{2}{cU}\, R\ell \oint_{\text{body}} w(\zeta) \frac{dz}{d\zeta}\, d\zeta \tag{11}$$

where $R\ell$ denotes the real part of the integral. From Eqs. (6), (7) it is seen that

$$C_L = -\frac{2}{cU} R\ell \, J \, .$$

The lift coefficient becomes after some manipulation

$$C_L = \pi \alpha \left[1 + (1 - \ell/c)^{-1/2} \right] \, . \tag{12}$$

For $\ell/c \ll 1$ the approximate result

$$C_L = 2\pi\alpha \left[1 + \frac{16\alpha^2}{K^2} + \ldots \right] \tag{12a}$$

is obtained.

Discussion

From the plot of $K/2\alpha$ vs bubble length in Fig. 3 it is seen that K/α has a minimum value when the cavity-chord ratio (ℓ/c) is about 0.74. The theory is thus limited to values of $\ell/c < 0.74$, otherwise it would be possible to have two different cavity lengths for the same cavitation number and angle of attack. Due to the restrictions of the thin airfoil assumptions, the theory is not likely to be much good for $\ell/c > 0.5$ anyway. Thus the thin airfoil theory fails for $\ell/c \doteq 1$ and to obtain more accurate information there a more exact method would have to be used, for example a hodograph theory. Tulin's result (Ref. 1) is plotted for $\alpha = 5.7^\circ$ in Fig. 3, and it also shows the singular behavior near $\ell/c = 1$.

The effect of cavitation number on lift coefficient is shown in Fig. 4. For K large and ℓ/c small the lift approaches the flat plate value. Some experimental lift measurements on a sharp-edged Karman-Trefftz profile are also plotted on Fig. 4.[3] The agreement with the flat plate theory is good for high cavitation numbers; however, the theory is inadequate for combinations of cavitation number and angles of attack that result in comparatively long cavities. That is, it does not predict the lift maximum found experimentally at the lower cavitation parameters. However, it is possible that real fluid phenomena are partially responsible for this result.

These results are, strictly speaking, applicable only to flat plate hydrofoils with cavitation starting at the sharp leading edge. The effect of cavitation on profile shapes with thickness is, no doubt, much more complicated.

References

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2. Wu, T. Yao-tsu, "A Free Streamline Theory for Two-Dimensional Fully Cavitated Hydrofoils", California Institute of Technology, Hydrodynamics Laboratory Report No. 21-17, July, 1955.

3. Balhan, J., "Metingen Aan Enige Bij Scheepsschroenen Gebruikelijke Profielen In vlokke Stroming Mit En Zonder Cavitatie", Ned. Scheepsbonrokundig Proefstation Te Wageningen, Publicatie No. 99, 1951.

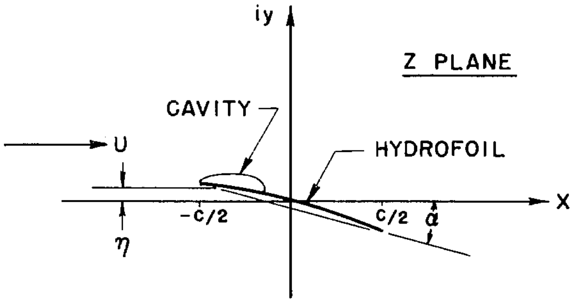

Fig. 1 - Hydrofoil with partial cavitation.

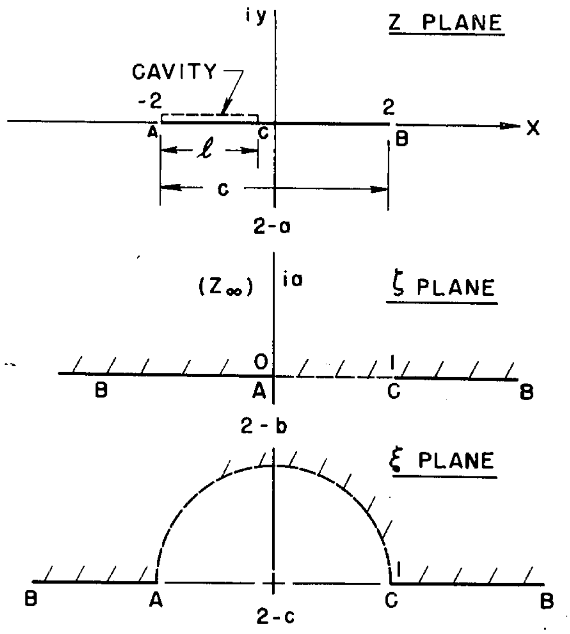

Fig. 2 - Representation of hydrofoil in the z- and auxiliary planes.

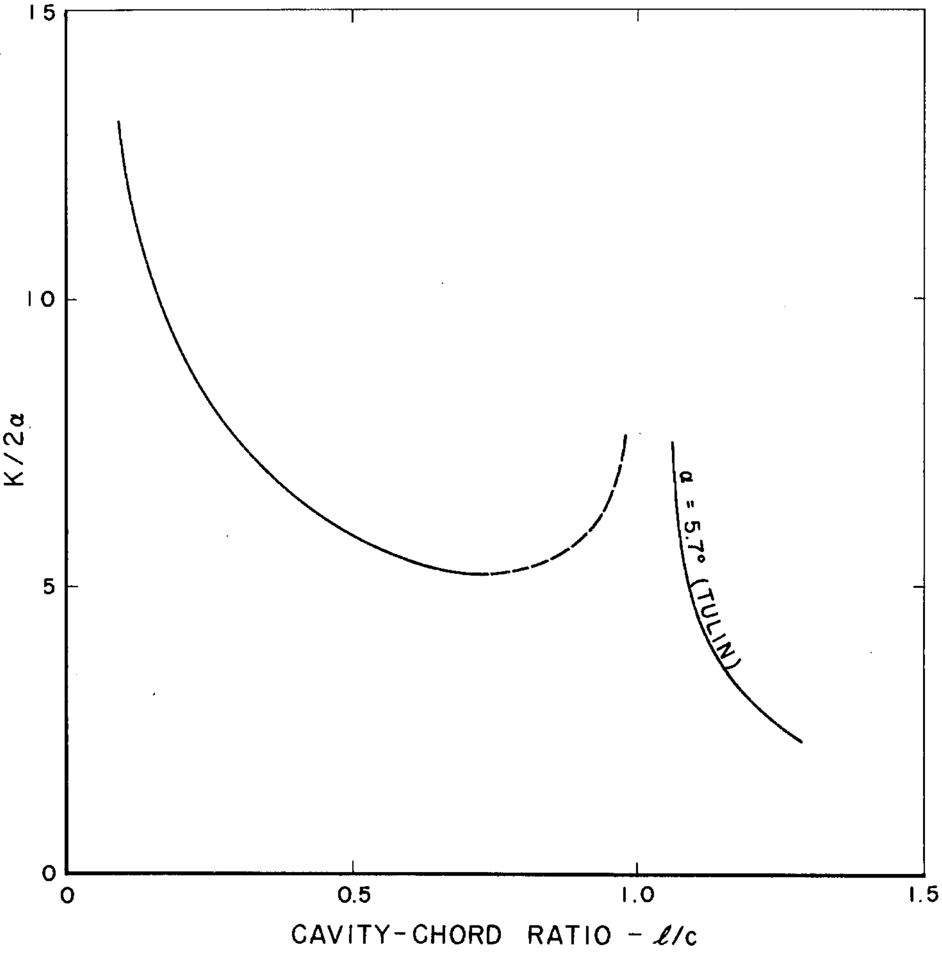

Fig. 3 - Cavity length in terms of cavitation number and angle of attack for a flat plate hydrofoil.

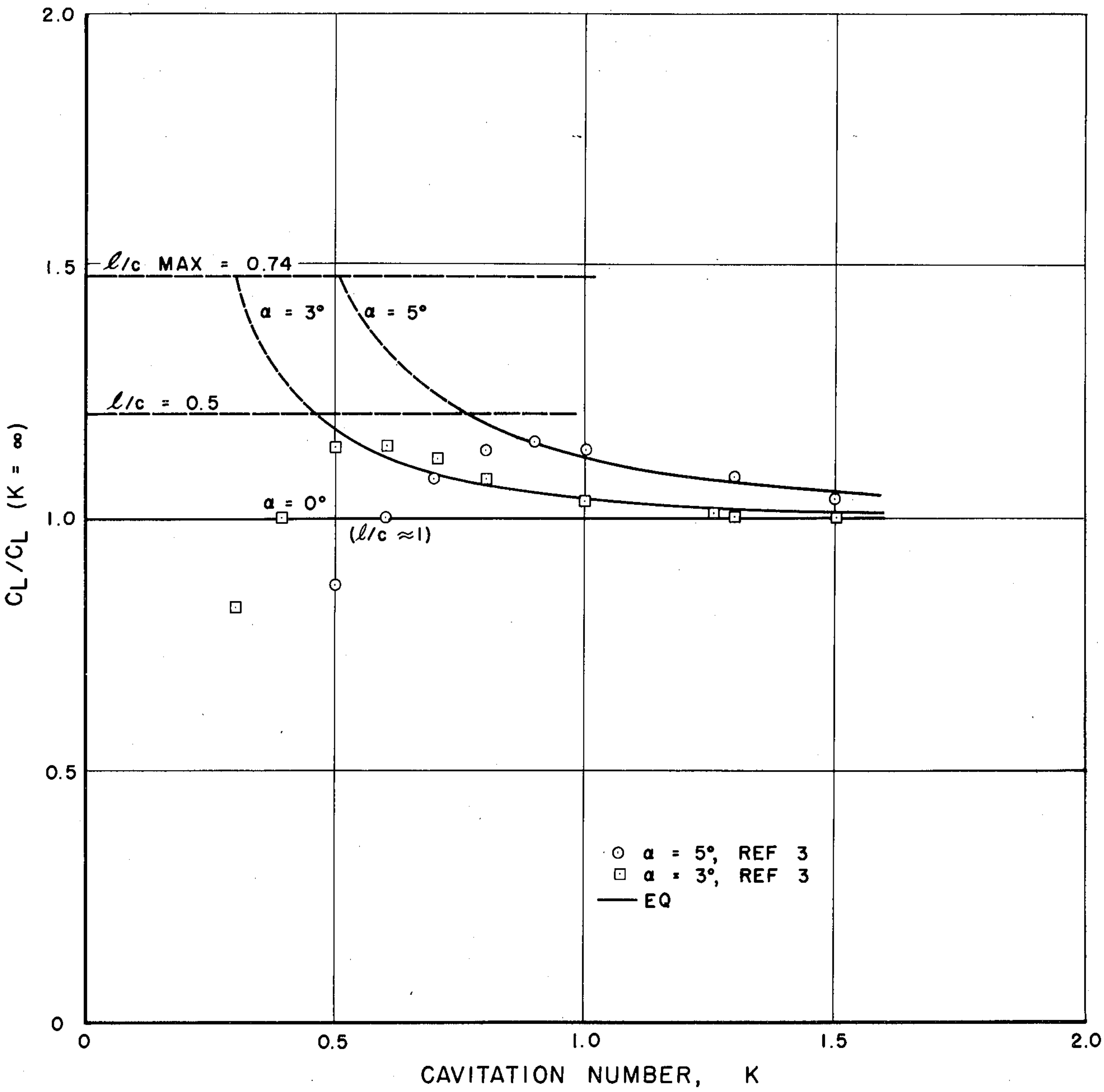

Fig. 4 - Ratio of cavitating to noncavitating lift coefficient vs cavitation number for a flat plate hydrofoil.

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